

HEAVY FLAVOURS FROM QCD SPECTRAL SUM RULES

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Abstract

We present a summary update of the QCD spectral sum rule (QSSR) results for the running and *perturbative pole* quark masses, the f_D and f_B leptonic decay constants, the heavy-to-light and heavy-to-heavy exclusive transition-form factors. Analytic expressions of these latter quantities are presented, which give a deeper understanding of their q^2 - and infinite mass-behaviours. A short comparison of the QSSR results with alternative approaches is done.

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1 QCD Spectral Sum Rules (QSSR)

QCD spectral sum rule (QSSR) à la SVZ [1] (for a recent review, see e.g. [2]) has shown since 15 years, its impressive ability for describing the complex phenomena of hadronic physics with the few universal “fundamental” parameters of the QCD Lagrangian (QCD coupling α_s , quark masses and vacuum condensates built from the quarks and/or gluon fields), without waiting for a complete understanding of the confinement problem. In the example of the two-point correlator:

$$\Pi_b(q^2) \equiv i \int d^4x e^{iqx} \langle 0 | \mathcal{T} J_b(x) (J_b(o))^\dagger | 0 \rangle, \quad (1)$$

associated to the generic hadronic current: $J_b(x) \equiv \bar{q}\Gamma b(x)$ of the q and b -quarks (Γ is a Dirac matrix which specifies the hadron quantum numbers), the SVZ-expansion reads:

$$\Pi_b(q^2) \simeq \sum_{D=0,2,\dots} \sum_{\dim O=D} \frac{C^{(J)}(q^2, M_b^2, \mu) \langle O(\mu) \rangle}{(M_b^2 - q^2)^{D/2}}, \quad (2)$$

where μ is an arbitrary scale that separates the long- and short-distance dynamics; $C^{(J)}$ are the Wilson coefficients calculable in perturbative QCD by means of Feynman diagrams techniques; $\langle O \rangle$ are the non-perturbative condensates of dimension D built from the quarks or/and gluon fields ($D = 0$ corresponds to the case of the naïve perturbative contribution). Owing to gauge invariance, the lowest dimension condensates that can be formed are the $D = 4$ light quark $m_q \langle \bar{\psi}\psi \rangle$ and gluon $\langle \alpha_s G^2 \rangle$ ones, where the former is fixed by the pion PCAC relation, whilst the latter is known to be $(0.07 \pm 0.01) \text{ GeV}^4$ from more recent analysis of the light [3] and heavy quark systems [2]. The validity of the SVZ-*postulate* has been understood formally, using renormalon techniques (absorption of the IR renormalon ambiguity into the definitions of the condensates, UV renormalon cannot induce some extra $1/M^2$ -terms not included in the OPE) [4, 5] and/or by building renormalization-invariant combinations of the condensates (Appendix of [6] and references therein). The SVZ expansion is phenomenologically confirmed from the unexpected accurate determination of the QCD coupling α_s and from a measurement of the condensates from semi-inclusive tau decays [6, 7]. The previous QCD information is transmitted to the data through the spectral function $\text{Im}\Pi_b(t)$ via the Källen–Lehmann dispersion relation (*global duality*) obeyed by the hadronic correlators, which can be improved from the uses of either a finite number of derivatives and finite values of q^2 (moment sum rules):

$$\mathcal{M}^{(n)} \equiv \frac{1}{n!} \frac{\partial^n \Pi_b(q^2)}{(\partial q^2)^n} \Big|_{q^2=0} = \int_{M_b^2}^{\infty} \frac{dt}{t^{n+1}} \frac{1}{\pi} \text{Im}\Pi_b(t), \quad (3)$$

or an infinite number of derivatives and infinite values of q^2 , but keeping their ratio fixed as $\tau \equiv n/q^2$ (Laplace or exponential sum rules):

$$\mathcal{L}(\tau, M_b^2) = \int_{M_b^2}^{\infty} dt \exp(-t\tau) \frac{1}{\pi} \text{Im}\Pi_b(t), \quad (4)$$

for $m_q = 0$. Non-relativistic versions of these two sum rules are convenient quantities to work with, in the large-quark-mass limit, after introducing the non-relativistic variables E and τ_N : $t \equiv (E + M_b)^2$ and $\tau_N \sim M_b \tau$. In the previous sum rules, the weight factors enhance the contribution of the lowest ground-state meson to the spectral integral, such that, the simple duality ansatz parametrization: “one narrow resonance” + “QCD continuum”, from a threshold t_c ,

gives a very good description of the spectral integral. The previous naïve parametrization has been tested successfully in the light-quark channel from the $e^+e^- \rightarrow I = 1$ hadron data and in the heavy-quark ones from the $e^+e^- \rightarrow \psi$ or Υ data, within a good accuracy. In principle, the pairs (n, t_c) , (τ, t_c) are free external parameters in the analysis, so that the optimal result should be insensitive to their variations. *Stability criteria*, which are equivalent to the variational method, state that the best results should be obtained at the minimas or at the inflexion points in n or τ , while stability in t_c is useful to control the sensitivity of the result in the changes of t_c -values. To these stability criteria can be added constraints from local duality FESR, which correlate the t_c -values to those of the ground state mass and coupling [8]. Stability criteria have also been tested in models such as the harmonic oscillator [9], where the exact and approximate solutions are known. However, though *I would personally expect* that the *true* result is obtained near the beginning of the t_c -stability region ¹ despite the fact that in some cases this value of t_c is larger than the phenomenological guessed position of the next radial excitation, one can *fairly* state that the *most conservative optimization criteria*, which include various types of optimizations in the literature, are the one obtained in the region, starting from the beginning of τ/n stability ², until the beginning of the t_c stability. One can *a posteriori* check that, at the stability point, where we have an equilibrium between the continuum and non-perturbative contributions, which are both small, the OPE is still convergent and the expansion certainly makes sense. The results which will be quoted below have been obtained within the previous *stability criteria*.

2 The heavy-quark-mass values

Many efforts have been devoted to the study of the quark masses [10]. Using the present *world average* value $\alpha_s(M_Z) = 0.118 \pm 0.006$ [11], the *first direct* determination of the running mass to two loops, from the Ψ and Υ systems, is [12]:

$$\begin{aligned}\overline{m}_c(M_c^{PT2}) &= (1.23_{-0.04}^{+0.02} \pm 0.03) \text{ GeV} \\ \overline{m}_b(M_b^{PT2}) &= (4.23_{-0.04}^{+0.03} \pm 0.02) \text{ GeV},\end{aligned}\tag{5}$$

where the errors are respectively due to α_s and to the gluon condensate. Using the relation [13]:

$$M_Q = \overline{m}_Q(M_Q^2) \left\{ 1 + \frac{4}{3} \left(\frac{\alpha_s}{\pi} \right) + K_Q \left(\frac{\alpha_s}{\pi} \right)^2 \right\},\tag{6}$$

where $K_b \simeq 12.4$, $K_c \simeq 13.3$ [14], one can transform this result into the *perturbative pole* mass and obtain, to two-:

$$\begin{aligned}M_c^{PT2} &= (1.42 \pm 0.03) \text{ GeV} \\ M_b^{PT2} &= (4.62 \pm 0.02) \text{ GeV},\end{aligned}\tag{7}$$

¹ This value of t_c is about the one fixed by FESR duality constraints. In this region, the result is certainly insensitive to the form of the continuum model.

²This corresponds in most of the cases to the so-called plateau often discussed in the literature, but in my opinion, the interpretation of this plateau as a sign of a good continuum model is not sufficient, in the sense that the flatness of the curve extends in the uninteresting high-energy region where the properties of the ground state are lost.

and three-loop accuracy:

$$\begin{aligned} M_c^{PT3} &= (1.62 \pm 0.07 \pm 0.03) \text{ GeV} \\ M_b^{PT3} &= (4.87 \pm 0.05 \pm 0.02) \text{ GeV}. \end{aligned} \quad (8)$$

It is informative to compare these values with the ones of the pole masses from HQET [15, 16] and non-relativistic sum rules to two loops [12]:

$$\begin{aligned} M_c^{NR} &= (1.45_{-0.03}^{+0.04} \pm 0.03) \text{ GeV} \\ M_b^{NR} &= (4.69_{-0.01}^{+0.02} \pm 0.02) \text{ GeV}, \end{aligned} \quad (9)$$

where one might interpret the small mass difference of about 70 MeV as the size of the renormalon effect into the pole mass. Indeed, an explicit resummation of the leading $(\beta\alpha_s)^n$ terms increases the previous two-loop estimate by about 100~200 MeV [5]. It also indicates that the three-loop value in Eq. (8) already gives a good estimate of the pole mass to all orders of PT. Eq. (8) compares quite well with the *dressed* mass $M_b^{nr} = (4.94 \pm 0.10 \pm 0.03) \text{ GeV}$, obtained from a non-relativistic higher-order Balmer formula based on a $\bar{b}b$ Coulomb potential [17] and with lattice calculations [18]. One can also use the previous results, in order to deduce the (non)-relativistic pole mass-difference of the b - and c -quarks both evaluated at M_b :

$$M_b - M_c \Big|_{p^2=M_b^2} = (3.54 \pm 0.05) \text{ GeV}. \quad (10)$$

Finally, the *lesson* which we can learn from the previous discussion is that one should be very careful in using the *numerical value* of the quark mass. Indeed, for consistency, one should first understand the definition of the mass used in the analysis and know to what loop-accuracy the analysis is done (for a recent compilation of the running quark masses, see e.g. [19]).

3 The decay constants and the B_B -parameter

The decay constants f_P of a pseudoscalar meson P are defined as:

$$(m_q + M_Q) \langle 0 | \bar{q}(i\gamma_5)Q | P \rangle \equiv \sqrt{2} M_P^2 f_P, \quad (11)$$

where in this normalization $f_\pi = 93.3 \text{ MeV}$. A lot of efforts have also been done for the estimate of these decay constants [2]. However, the most pertinent result has been obtained, *for the first time*, in [20]:

$$f_D \approx f_B \simeq 1.4 f_\pi, \quad (12)$$

which clearly shows a violation of the Infinite Mass Effective Theory (IMET) $1/\sqrt{M_b}$ -scaling law. Several obscure and unjustified criticisms have been addressed later on in order to discredit such a result, but a numerical estimate of the $1/M_b$ -corrections complemented by the lattice results and by the analytic estimate from HQET and semi-local duality has provided a better understanding of the *unexpected* result in Eq. (12). Indeed, using the present update *best* estimate from the Laplace sum rule [21]:

$$\begin{aligned} f_D &\simeq (1.35 \pm 0.04 \pm 0.06) f_\pi \\ f_B &\simeq (1.49 \pm 0.06 \pm 0.05) f_\pi, \end{aligned} \quad (13)$$

consistent with the value of the relativistic two-loop pole mass given previously, and using the value of the decay constant in the static limit [16, 22]:

$$f_B^\infty \simeq (1.98 \pm 0.31)f_\pi, \quad (14)$$

the $1/M_b$ -corrections are found to be [23, 24, 15, 22]:

$$f_B \sqrt{M_b} \simeq (0.33 \pm 0.06) \text{GeV}^{3/2} \alpha_s^{1/\beta_1} \left\{ 1 - \frac{2}{3} \frac{\alpha_s}{\pi} - \frac{(A \simeq 1.1 \text{ GeV})}{M_b} + \frac{(B \simeq 0.7 \text{ GeV}^2)}{M_b^2} \right\} \quad (15)$$

which, one can *qualitatively* compare with the one obtained from the analytic expression of the moments or from the semilocal duality sum rule leading to the *interpolating formula* [25]:

$$f_B \sqrt{M_b} \approx \frac{E_c^{3/2}}{\pi \sqrt{2}} \alpha_s^{1/\beta_1} \left(\frac{M_b}{M_B} \right)^{3/2} \left\{ 1 - \frac{2}{3} \frac{\alpha_s}{\pi} + \frac{3}{88} \frac{E_c^2}{M_b^2} - \frac{\pi^2}{2} \frac{\langle \bar{u}u \rangle}{E_c^3} + \dots \right\}, \quad (16)$$

and gives for $E_c \simeq 1.3 \text{ GeV}$:

$$\begin{aligned} A &\approx \frac{3}{2}(M_B - M_b) \simeq 1 \text{ GeV}, \\ B &\approx \frac{3}{88} E_c^2 - \frac{9}{8}(M_B - M_b)^2 \simeq 0.5 \text{ GeV}^2, \end{aligned} \quad (17)$$

The $SU(3)$ -breaking effects to these decay constants have also been estimated *analytically* to be [21]:

$$\frac{f_{D_s}}{f_D} \simeq \frac{f_{B_s}}{f_B} \simeq (1.16 \pm 0.04)f_\pi, \quad (18)$$

which is in good agreement with the range of values obtained from different lattice groups [26]. The corresponding value of $f_{D_s} \simeq (1.55 \pm 0.10)f_\pi$ is still compatible (within the errors) with the recent (indirect) measurements from WA75 and CLEOII (see e.g.[27]), which need to be tested from direct (though difficult) measurements of the leptonic widths. However, independently of the charm quark mass-value, which affects strongly the value of f_D [21] (f_D increases for decreasing M_c), a value of f_{D_s} larger than the *rigorous* upper bound of $2.14f_\pi$ deduced from [28] and Eq. (18) is unlikely from the QSSR approach within the standard SVZ-expansion. We have also tested the validity of the vacuum saturation for the B_B -parameter, and we found that the radiative corrections for the non-factorized correlators are quite small (less than 15%), from which we deduce [29]:

$$B_B \simeq 1 \pm 0.15. \quad (19)$$

4 Heavy-to-light transition-form factors

One can extend the analysis done for the two-point correlator to the more complicated case of three-point function, in order to study the form factors related to the $B \rightarrow \pi(\rho)l\nu$ and

$B \rightarrow K^*\gamma$ rare decays. In so doing, one can consider the generic three-point function:

$$V \equiv - \int d^4x d^4y e^{i(p'x - py)} \langle 0 | \mathcal{T} J_L(x) O(0) J_B^\dagger(y) | 0 \rangle, \quad (20)$$

where J_L , J_B are the currents of the light and B mesons; O is the weak operator specific for each process (penguin for the $K^*\gamma$, weak current for the semileptonic); $q \equiv p - p'$ is the momentum transfer. The vertex obeys a double dispersion relation, which can be improved into the *unique*³ hybrid sum rule (HSR) [23, 30]:

$$\mathcal{H}(n, \tau') = \frac{1}{\pi^2} \int_{M_b^2}^{\infty} \frac{ds}{s^{n+1}} \int_0^{\infty} ds' e^{-\tau' s'} \text{Im} V(s, s'), \quad (21)$$

corresponding to a finite number n ($n \approx 1 - 2$ in the present processes) of the moments for the heavy-quark channel and to the Laplace for the light one. We have studied analytically the different form factors⁴ entering the previous processes [31], and we found that they are dominated *universally*, for $M_b \rightarrow \infty$, by the light-quark-condensate contribution as:

$$F(0) \sim \frac{\langle \bar{d}d \rangle}{f_B} \left\{ 1 + \frac{\mathcal{I}_F}{M_b^2} \right\}, \quad (22)$$

where \mathcal{I}_F is the integral from the perturbative triangle graph, which is constant as $t_c'^2 E_c / \langle \bar{d}d \rangle$ (t_c' and E_c are the continuum thresholds of the light and b quarks) for large values of M_b . Unlike the case of f_B , where the perturbative graph and the $\langle \bar{q}q \rangle$ condensate are of the same order in M_b , the present dominance of the $\langle \bar{q}q \rangle$ condensate allows a good separation of the lowest ground state contribution from the radial excitation. It also indicates that at $q^2 = 0$ and to leading order in $1/M_b$, all form factors behave like $\sqrt{M_b}$, although, in most cases, the coefficients of $1/M_b$ due mainly to f_B and of $1/M_b^2$ due to the perturbative graph are large⁵. In the particular case of $B \rightarrow \pi l \nu$, the form factor can be simply written to leading order [33]:

$$f_+(0) \simeq \left(\frac{1}{4f_\pi} \right) \left(\frac{f_\pi^2}{f_B} \right) \simeq 0.15, \quad (23)$$

to be compared with the numerical estimate 0.25 (the factor f_π^2 reflects the off-shellness of the pion) and to the pion coupling to hadron pairs [2]. The study of the q^2 behaviours of the form factors shows that, with the exception of the A_1 form factor, their q^2 dependence is only due to the non-leading $1/M_b^2$ perturbative graph, so that for $M_b \rightarrow \infty$, these form factors remain constant from $q^2 = 0$ to q_{max}^2 and have a weaker q^2 -dependence (polynomial in q^2 that can be resummed), than the pole model at finite M_b (here the value of pole mass which fits the form factors is about 5–6 GeV [34] (see also [35])). The resulting M_b behaviour at q_{max}^2 is the one expected from the heavy quark symmetry. The situation for the A_1 is drastically different from the other ones and from the pole parametrization. Here the Wilson coefficient of the $\langle \bar{d}d \rangle$ condensate contains a q^2 dependence with a *wrong* sign and reads:

$$A_1(q^2) \sim \frac{\langle \bar{d}d \rangle}{f_B} \left\{ 1 - \frac{q^2}{M_b^2} \right\}, \quad (24)$$

³The *popular* double exponential sum rule is not appropriate here as in this sum rule the OPE blows up for $M_b \rightarrow \infty$.

⁴In the standard notations, the relevant form factors are f_+ , (A_1 , A_2 , V) for $B \rightarrow \pi(\rho) l \nu$ and F_1 for $B \rightarrow K^*\gamma$ decays.

⁵This feature also indicates that it is *dangerous* to extrapolate the M_b -dependence obtained at the c -quark mass to higher quark mass values.

which, for $q_{max}^2 \equiv (M_B - M_\rho)^2$, gives the expected $1/\sqrt{M_b}$ behaviour. This result also explains the numerical observation in [34] (similar conclusions using alternative approaches have also been reached in [36]). Numerically, we obtain at $q^2 = 0$, the value of the $B \rightarrow \rho(K^*)\gamma$ form factors:

$$F_1^{B \rightarrow \rho} \simeq 0.27 \pm 0.03, \quad \frac{F_1^{B \rightarrow K^*}}{F_1^{B \rightarrow \rho}} \simeq 1.14 \pm 0.02, \quad (25)$$

which leads to the branching ratio $(4.5 \pm 1.1) \times 10^{-5}$, in perfect agreement with the CLEO data, while the numerical agreement with the estimate in [37] from light cone sum rule (for a criticism on the unreliability for the construction of the hadronic wave functions on the light-cone, see e.g. [38]) may only be due to the importance of the perturbative contribution at this scale⁶. For the semileptonic decays, a determination of the ratios of the form factors gives a more precise prediction [30] than from a direct estimate of the absolute values:

$$\begin{aligned} \frac{A_2(0)}{A_1(0)} &\simeq \frac{V(0)}{A_1(0)} \simeq 1.11 \pm 0.01, \\ \frac{A_1(0)}{F_1^{B \rightarrow \rho}(0)} &\simeq 1.18 \pm 0.06, \quad \frac{A_1(0)}{f_+(0)} \simeq 1.40 \pm 0.06. \end{aligned} \quad (26)$$

Combining these results with the “world average” value of $f_+(0) = 0.25 \pm 0.02$ and the one of $F_1^{B \rightarrow \rho}(0)$, one can deduce the rate and polarization:

$$\begin{aligned} \Gamma_\pi &\simeq (4.3 \pm 0.7) |V_{ub}|^2 \times 10^{12} \text{ s}^{-1} \quad \frac{\Gamma_\rho}{\Gamma_\pi} \simeq 0.9 \pm 0.2 \\ \frac{\Gamma_+}{\Gamma_-} &\simeq 0.20 \pm 0.01 \quad \alpha \equiv 2 \frac{\Gamma_L}{\Gamma_T} - 1 \approx -0.6. \end{aligned} \quad (27)$$

These precise results may indicate that, V_{ub} can be reached with a good accuracy from the exclusive modes. The non-pole behaviour of A_1 affects strongly the different estimates in Eq. (27), in particular the ones of Γ_ρ/Γ_π and α , such that a firm prediction of these quantities needs an improved good control of the q^2 -dependence of the corresponding form factors. We extend the previous analysis to the estimate of the $SU(3)$ breaking in the ratio of the form factors:

$$R_P \equiv f_+^{P \rightarrow K}(0)/f_+^{P \rightarrow \pi}(0), \quad (28)$$

where $P \equiv \bar{B}, D$. Its analytic expression is given in [32] and leads to the numerical result:

$$R_B = 1.007 \pm 0.020 \quad R_D = 1.102 \pm 0.007, \quad (29)$$

which is typically of the same size as the one of f_{D_s} and of the $B \rightarrow K^*\gamma$ discussed before and reinforces the credibility of the present estimate. However, it is quite surprising that using the previous value of R_D into the present value of the CLEO data [40]:

$$\frac{Br(D^+ \rightarrow \pi^0 l \nu)}{Br(D^+ \rightarrow \bar{K}^0 l \nu)} = (8.5 \pm 2.7 \pm 1.4)\%, \quad (30)$$

one deduces:

$$V_{cd}/V_{cs} = 0.322 \pm 0.056, \quad (31)$$

which is much larger than the value 0.226 ± 0.005 derived from the unitarity of the CKM matrix. This *apparent* discrepancy needs a further measurement of the previous process before a firm conclusion can be drawn (recall that MARKIII data [41] would imply a value 0.25 ± 0.15 compatible within the errors with Eq. (31).).

⁶Lattice results on the radiative and semi-leptonic decays are reported in [39]

5 $B^*B\pi(\gamma)$ couplings and $D^* \rightarrow D\pi(\gamma)$ widths

As has been studied recently in [42], the previous processes are very similar to the other heavy-to-heavy transitions as they are dominated by the perturbative graph contributions. The non-leading $1/M_b$ corrections for the radiative decays are large as they come mainly from the heavy quark component of the electromagnetic current. This contribution is essential for explaining the large charge dependence in the observed radiative decay widths. For the B^* meson, our predictions *without any free parameters* are:

$$g_{B^*B\pi} \simeq 14 \pm 5 \quad \Gamma_{B^{*-} \rightarrow B^- \gamma} / \Gamma_{B^{*-} \rightarrow B^- \gamma} \simeq 2.5, \quad (32)$$

where the latter indicates a large isospin violation, which deviates strongly from the naïve static limit e_u^2/e_d^2 expectation, therefore showing the importance of the $1/M_b$ corrections in this channel. For the D^* -one, we find:

$$\Gamma_{D^{*-} \rightarrow D^0 \pi^-} \simeq 1.54 \Gamma_{D^{*0} \rightarrow D^0 \pi^0} \simeq (8 \pm 5) \text{ keV}, \quad (33)$$

while:

$$\begin{aligned} \Gamma_{D^{*-} \rightarrow D^- \gamma} &\simeq (0.09^{+0.40}_{-0.07}) \text{ keV} \\ \Gamma_{D^{*0} \rightarrow D^0 \gamma} &\simeq (3.7 \pm 1.2) \text{ keV}. \end{aligned} \quad (34)$$

The resulting total widths $\Gamma_{D^{*-} \rightarrow \text{all}} \simeq (12 \pm 7) \text{ keV}$ and $\Gamma_{D^{*0} \rightarrow \text{all}} \simeq (11 \pm 4) \text{ keV}$ are much smaller than the present experimental upper limits. Improved measurements of these widths in the next τ -charm factory machine should provide a decisive test of the predictions given here and should also help to clarify the disagreements among the present theoretical predictions.

6 Slope of the Isgur–Wise function and V_{cb}

From the QSSR expression of the *universal* Isgur–Wise function, to leading order in $1/M_b$ [15]:

$$\begin{aligned} \zeta_{phys}(y \equiv vv') &= \left(\frac{2}{1+y} \right)^2 \left\{ 1 + \frac{\alpha_s}{\pi} f(y) \right. \\ &\quad \left. - \langle \bar{d}d \rangle \tau^3 g(y) + \langle \alpha_s G^2 \rangle \tau^4 h(y) + g \langle \bar{d}Gd \rangle \tau^5 k(y) \right\}, \end{aligned} \quad (35)$$

where τ is the Laplace sum rule variable and f , h and k are analytic functions of y . From this expression, one can derive the analytic form of the slope of the IW-function [43]:

$$\zeta'_{phys}(y=1) \simeq -1 + \delta_{pert} + \delta_{NP} \simeq -1 \pm 0.02, \quad (36)$$

where at the τ -stability region:

$$\delta_{pert} \simeq -\delta_{NP} \simeq -0.04, \quad (37)$$

which shows the near-cancellation of the non-leading corrections, and we have added a generous 50% error of 0.02 for the correction terms. This result is in agreement with the improved bound of Taron–de Rafael on the slope of the form factor [44]:

$$F'(vv' = 1) \geq -1.5, \quad (38)$$

based on the analyticity, the positivity and a mapping technology of the elastic b -number form factor F defined as:

$$\langle B(p') | \bar{b} \gamma^\mu b | B(b) \rangle = (p + p')^\mu F(q^2), \quad (39)$$

and normalized as $F(0) = 1$ in the large mass limit $M_B \simeq M_D$. The inclusion of the effects of the Υ states below $\bar{B}B$ thresholds by using the sum of the $\Upsilon \bar{B}B$ couplings of 0.34 ± 0.02 from QSSR improves slightly this bound to:

$$F'(vv' = 1) \geq -1.34. \quad (40)$$

Using the relation of the form factor with the slope of the Isgur–Wise function, which differs by $-16/75 \log \alpha_s(M_b)$ [45], one can deduce the final bound:

$$\zeta'(1) \geq -1.04. \quad (41)$$

The QSSR results are in good agreement with some other significant estimates given in the literature.

Let us now discuss the effects due to the $1/M$ corrections, which can be done in two ways: By calculating the $1/M_b$ corrections within HQET, [46] (resp. [47]) obtains:

$$\eta_A \zeta(1) = 0.93 \pm 0.03 \text{ (resp } 0.89 \pm 0.03) . \quad (42)$$

Here, the model-dependence enters when extrapolating the data at $y = 1$, and leads to:

$$V_{cb} \simeq (39.9 \pm 2.9) \times 10^{-3} \text{ (resp } (41.7 \pm 3) \times 10^{-3}), \quad (43)$$

Alternatively, one can use the value of the form factor at $q^2 = 0$ ($y = 1.5$) from the sum rule in the *full theory* [30]:

$$F(1.5) = 0.53 \pm 0.09 \quad (44)$$

and the data in the *whole range* of y in order to deduce the slope:

$$\rho^2 \equiv -\zeta' \simeq 0.76 \pm 0.2, \quad (45)$$

from a linear parametrization of the form factor. The model dependence enters in this analysis through the curvature of the form factor. The main advantage of this approach is that it does not rely on the previous theoretical conflict at $y = 1$. Using the different data from CLEOII, ARGUS and ALEPH [48], we obtain the average:

$$V_{cb} \simeq (39.9 \pm 1.2 \pm 1.4) \times 10^{-3}, \quad (46)$$

where the first error is from the data, and the second one is from the type of model-parametrizations. This result is in good agreement with the one in Eq. (43) and from the inclusive decays.

7 Conclusion

We have shortly presented different results from QCD spectral sum rules in the heavy-quark sector, which are useful for further theoretical studies of the B -physics and which complement the results from alternative non-perturbative approaches. From the experimental point of view, QSSR predictions agree with available data, but they also lead to some new features, which need to be tested in forthcoming experiments.

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